# The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS & MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2019-2020

**MECHANICS OF SOLIDS**

Time allowed AS PER SUBMISSION DEADLINE PUBLISHED ON MOODLE

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*Open-book take-home examination*

***Answer ALL questions***

*You must submit a single pdf document, produced in accordance with the guidelines provided on take-home examinations, that contains all of the work that you wish to have marked for this open-book examination. Your submission file should be named in the format '*[Student ID]_[Module Code].pdf*'.*

*Write your student ID number at the top of each page of your answers.*

*This work must be carried out and submitted as described on the Moodle page for this module. All work should have been submitted via Moodle by the due date.*
***Work submitted after the deadline will be subject to penalty.***

*No teaching enquiries will be answered by staff during the assessment period Monday 18th May to Friday 12th June 2020 and no questions should be raised by students. If you believe there is a misprint note it in your submission and answer the question as written. Contact* SS-Programmes-UPE@exmail.nottingham.ac.uk *for any support.*

***Plagiarism, false authorship and collusion are serious academic offences** as defined in the University's Academic Misconduct Policy and will be dealt with in accordance with the University's Academic Misconduct Procedures. The work submitted by students must be their own and you must declare that you understand the meaning of academic misconduct and have not engaged in it during the production of your work.*

**ADDITIONAL MATERIAL:** Formula sheet

**SECTION A**

Answer ALL questions in this section

1. The translation of the yield surface shown in Fig. Q1 represents

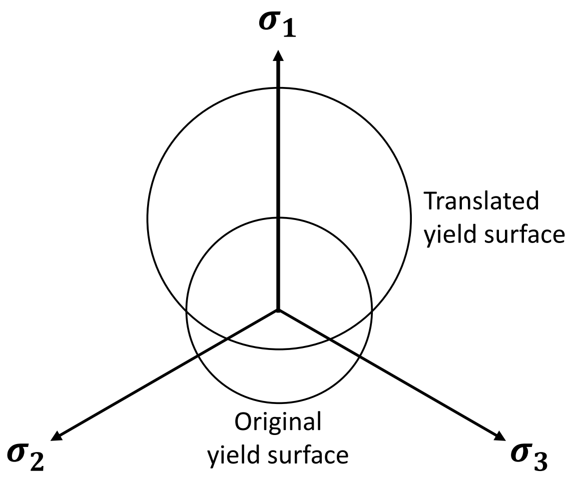


**Fig. Q1**

**A.** elastic-perfectly-plastic material behaviour
**B.** isotropic hardening
**C.** linear softening
**D.** kinematic hardening
**E.** Mixed isotropic and kinematic hardening

[2]

2. In the prediction of fatigue life, the Gerber curve is:

**A.** More conservative than the Goodman line
**B.** Also known as the Goodman line
**C.** Less conservative than the Goodman line
**D.** The same as the Goodman line in terms of conservatism
**E.** Not comparable

[2]

3. The following expression describes the strain energy in a beam:

$$U = \frac{P^3}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + \frac{\pi R^3}{3} + 2\pi R^2\right)$$

What is the deflection at the position of and in the direction of load, $P$?

**A.** $u = \frac{3P^2}{EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + \frac{\pi R^3}{3} + 2\pi R^2\right)$

**B.** $U = \frac{P^3}{EI}\left(\frac{L^2}{8} + \frac{3\pi L^4 R}{48} + \left(\frac{\pi R^3}{3} + 2\pi R^2\right)L\right)$

**C.** $u = \frac{P^4}{4EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + \frac{\pi R^3}{3} + 2\pi R^2\right)$

**D.** $u = \frac{P^3}{EI}\left(\frac{3\pi L^2 R}{4} + \frac{\pi R^3}{3} + 2\pi R^2\right)$

**E.** $U = \frac{P^4}{4EI}\left(\frac{L}{4} + \frac{3\pi L^3 R}{12} + \frac{\pi R^3}{3} + 2\pi R^2\right)$

[2]

4. If, from Macauley's method a beam under bending has the following 2nd order differential equation:

$$EI\frac{d^2y}{dx^2} = R_A x + M_O\langle x-2\rangle^0 - P\langle x-4\rangle - q\frac{\langle x-6\rangle^2}{2}$$

What is the corresponding expression for slope of the beam?

**A.** $\frac{dy}{dx} = \frac{1}{EI}\left(R_A\frac{x^2}{2} + M_O\langle x-2\rangle - P\frac{\langle x-4\rangle^2}{2} - q\frac{\langle x-6\rangle^3}{6} + A\right)$

**B.** $y = \frac{1}{EI}\left(R_A\frac{x^3}{6} + M_O\frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24} + Ax + B\right)$

**C.** $EI\frac{dy}{dx} = R_A\frac{x^2}{2} + M_O\langle x-2\rangle - P\frac{\langle x-4\rangle^2}{2} - q\frac{\langle x-6\rangle^3}{6}$

**D.** $\frac{dy}{dx} = \frac{1}{EI}\left(R_A\frac{x^3}{6} + M_O\frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24} + A\right)$

**E.** $EIy = R_A\frac{x^3}{6} + M_O\frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24}$

[2]

5. What is the corresponding expression for deflection of the beam from Q4?

**A.** $y = \frac{1}{EI}\left(R_A \frac{x^3}{6} + M_O \frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24} + Ax + B\right)$

**B.** $\frac{dy}{dx} = \frac{1}{EI}\left(R_A \frac{x^2}{2} + M_O \langle x-2\rangle - P\frac{\langle x-4\rangle^2}{2} - q\frac{\langle x-6\rangle^3}{6} + A\right)$

**C.** $EI\frac{dy}{dx} = R_A \frac{x^2}{2} + M_O \langle x-2\rangle - P\frac{\langle x-4\rangle^2}{2} - q\frac{\langle x-6\rangle^3}{6}$

**D.** $\frac{dy}{dx} = \frac{1}{EI}\left(R_A \frac{x^3}{6} + M_O \frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24} + A\right)$

**E.** $EIy = R_A \frac{x^3}{6} + M_O \frac{\langle x-2\rangle^2}{2} - P\frac{\langle x-4\rangle^3}{6} - q\frac{\langle x-6\rangle^4}{24}$

[2]

6. Fig. Q6 shows a schematic of a waveform for stress-controlled fatigue testing, where:

i. $S_{\mathrm{min}}$ = Minimum Stress
ii. $S_{\mathrm{max}}$ = Maximum Stress
iii. $S_{\mathrm{mean}}$ = Mean Stress
iv. $S_{\mathrm{range}}$ = Stress Range
v. $S_{\mathrm{amp}}$ = Stress Amplitude

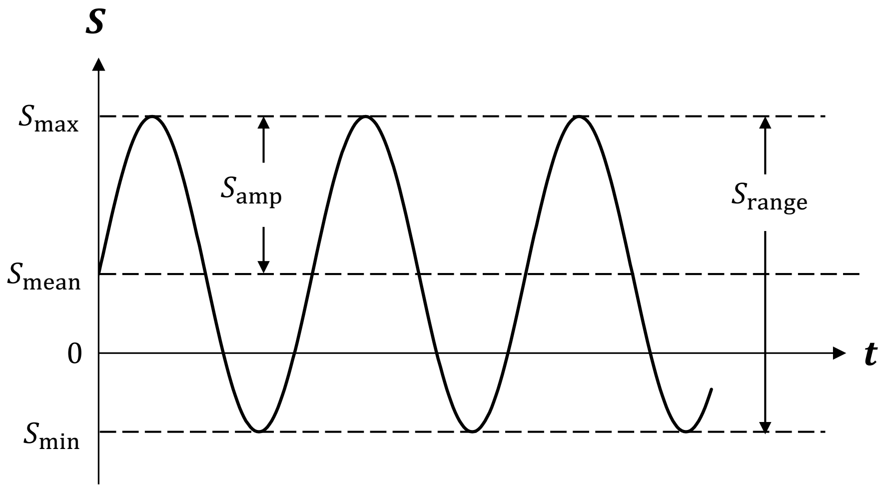


**Fig. Q6**

i to v are labelled correctly in Fig Q6.

**A.** True
**B.** -
**C.** -
**D.** -
**E.** False

[2]

7. Fig. Q7 shows a beam cross section and the position of the neutral axis for a particular loading condition. If position E represents the position of the maximum tensile bending stress, which position represents the position of maximum compressive bending stress?

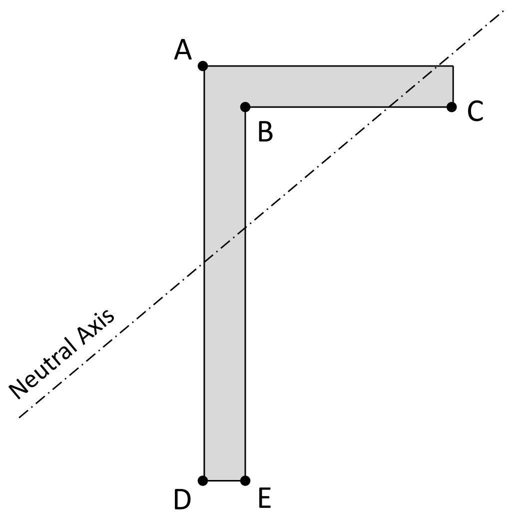


**Fig. Q7**

**A.** A
**B.** B
**C.** C
**D.** D
**E.** E

[2]

8. For a beam for which an expression for bending moment, $M$, can be derived, and deflection is measured positive upwards:

**A.** $EI\frac{dy}{dx} = -M$

**B.** $EI\frac{d^2y}{dx^2} = M$

**C.** $EI\frac{dy}{dx} = M$

**D.** $EI\frac{d^2y}{dx^2} = -M$

**E.** $EI\frac{d^2y}{dx^2} = M^2$

[2]

9. For a beam loaded along a principal axis:

**A.** The deflections in the loading direction and the perpendicular direction are always equal.
**B.** The deflection in the loading direction is always double the deflection in the perpendicular direction.
**C.** The beam cannot deflect.
**D.** The deflection in the perpendicular direction to the loading direction is always zero.
**E.** The deflection in the loading direction is always zero.

[2]

10. Increasing the length of a beam which is loaded along its length:

**A.** Eliminates the concern of buckling
**B.** Has no effect on the load required to cause buckling
**C.** Means the beam will definitely buckle
**D.** Reduces the load required to cause buckling
**E.** Increases the load required to cause buckling

[2]

11. What is the value of the maximum in-plane shear stress for the 2D plane-stress element shown in Fig. Q11?

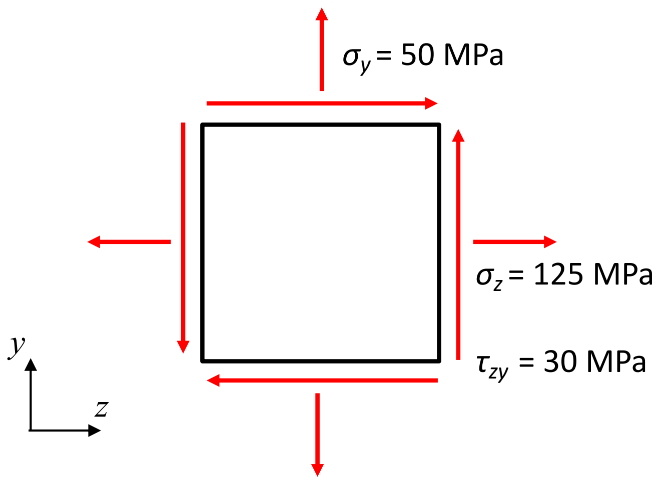


**Fig. Q11**

**A.** 62.5 MPa
**B.** 87.5 MPa
**C.** 48.0 MPa
**D.** 110.5 MPa
**E.** 92.5 MPa

[2]

12. Two cylinders of equal length have the following dimensions:

i. 12 mm bore and 24.03 mm outside diameter (steel: E = 210 GPa, $\nu$ = 0.3, $\alpha$ = 12 × $10^{-6}$ °$C^{-1}$)
ii. 24 mm bore and 44 mm outside diameter (bronze: E = 100 GPa, $\nu$ = 0.3, $\alpha$ = 17 × $10^{-6}$ °$C^{-1}$)

The larger cylinder is heated, placed around and allowed to shrink onto the smaller cylinder. What is the minimum temperature increase required to allow assembly?

**A.** 37 °C
**B.** 74 °C
**C.** 104 °C
**D.** 52 °C
**E.** 40 °C

[2]

13. The axial stiffness for a 0.3 m long, 1D truss element made of steel (E = 208 GPa) with a rectangular cross-section 15 mm x 30 mm is:

**A.** 3.12 x $10^5$ N/m
**B.** 4.66 x $10^8$ MN/m
**C.** 252 MN/m
**D.** 112 x $10^6$ N/m
**E.** 312 MN/m

[2]

14. A bar with a rectangular cross-section (b x d) of 25 mm x 40 mm subjected to a vertical shear force of 40 kN, what is the value of maximum shear stress in the bar?

**A.** 76.8 MPa
**B.** 153.6 MPa
**C.** 120 MPa
**D.** 60 MPa
**E.** 40 MPa

[2]

15. A rotor disc with an external diameter of 0.4 m and has a 0.05 m diameter hole bored along its axis is rotated at an angular velocity of 3000 rpm. What is the value of hoop stress at the bore? ($\rho$ = 8000 $kgm^{-3}$, $\nu$ = 0.3)

**A.** -26 MPa
**B.** 1.22 x $10^9$ Pa
**C.** -1.22 x $10^9$ Pa
**D.** 52 MPa
**E.** 26 MPa

[2]

16. A 25 mm diameter bar is subjected to a torque of 500 Nm and an axial load of 40 kN, the maximum principal stress on a 2D plane stress element on the surface of the bar is:

**A.** 163 MPa
**B.** 81.5 MPa
**C.** 168 MPa
**D.** 209 MPa
**E.** -127 MPa

[2]

17. From a design point-of-view, when compared to the von Mises yield criterion, the Tresca yield criterion is:

**A.** More conservative
**B.** -
**C.** -
**D.** -
**E.** Less conservative

[2]

18. The feature labelled A and highlighted in Fig. Q18 is the:

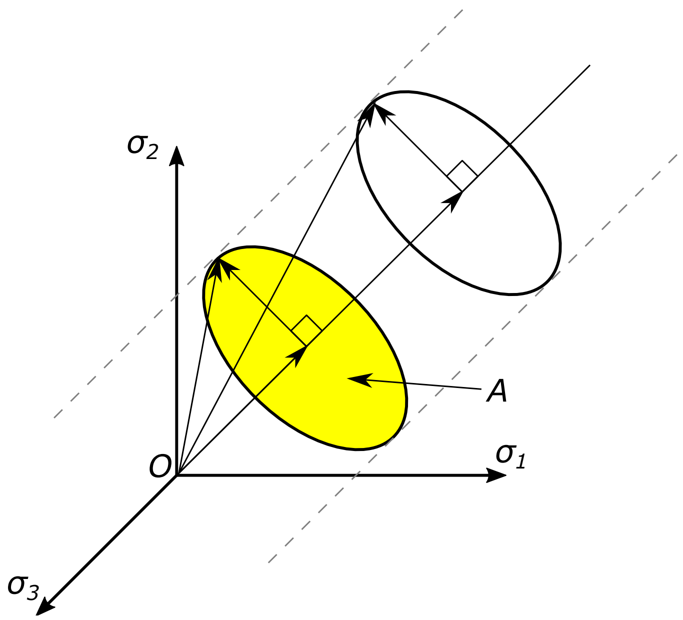


**Fig. Q18**

**A.** Hydrostatic Stress
**B.** Deviatoric Stress
**C.** $\pi$-plane
**D.** Deviatoric Plane
**E.** Principal Stress

[2]

19. The stiffness matrix for the element AB in the structure shown in Fig. Q19,

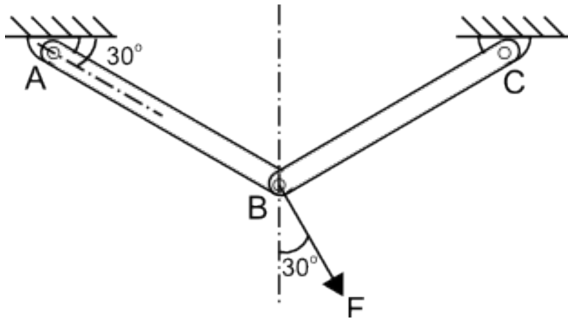


**Fig. Q19**

where the stiffness matrix of a truss element is given by:

$$[k_e] = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

and θ is the angle from the horizontal axis is:

**A.** $$[k_{AB}] = \left(\frac{AE}{L}\right)\begin{bmatrix} 0.75 & -\sqrt{3}/4 & -0.75 & \sqrt{3}/4 \\ -\sqrt{3}/4 & 0.25 & \sqrt{3}/4 & -0.25 \\ -0.75 & \sqrt{3}/4 & 0.75 & -\sqrt{3}/4 \\ \sqrt{3}/4 & -0.25 & -\sqrt{3}/4 & 0.25 \end{bmatrix}$$

**B.** $$[k_{AB}] = \left(\frac{AE}{L}\right)\begin{bmatrix} 0.75 & \sqrt{3}/4 & -0.75 & -\sqrt{3}/4 \\ \sqrt{3}/4 & 0.25 & -\sqrt{3}/4 & -0.25 \\ -0.75 & -\sqrt{3}/4 & 0.75 & \sqrt{3}/4 \\ -\sqrt{3}/4 & -0.25 & \sqrt{3}/4 & 0.25 \end{bmatrix}$$

**C.** $$[k_{AB}] = \left(\frac{AE}{L}\right)\begin{bmatrix} \sqrt{3}/4 & -0.75 & -\sqrt{3}/4 & 0.75 \\ -0.75 & 0.25 & 0.75 & -0.25 \\ -\sqrt{3}/4 & 0.75 & \sqrt{3}/4 & -0.75 \\ 0.75 & -0.25 & -0.75 & 0.25 \end{bmatrix}$$

**D.** $$[k_{AB}] = \left(\frac{AE}{L}\right)\begin{bmatrix} -\sqrt{3}/4 & -0.75 & \sqrt{3}/4 & 0.75 \\ -0.75 & -0.25 & 0.75 & 0.25 \\ \sqrt{3}/4 & 0.75 & -\sqrt{3}/4 & -0.75 \\ 0.75 & 0.25 & -0.75 & -0.25 \end{bmatrix}$$

**E.** $$[k_{AB}] = \left(\frac{AE}{L}\right)\begin{bmatrix} -0.75 & -\sqrt{3}/4 & 0.75 & \sqrt{3}/4 \\ -\sqrt{3}/4 & -0.25 & \sqrt{3}/4 & 0.25 \\ 0.75 & \sqrt{3}/4 & -0.75 & -\sqrt{3}/4 \\ \sqrt{3}/4 & 0.25 & -\sqrt{3}/4 & -0.25 \end{bmatrix}$$

[2]

20. A shaft, made of a material with $\sigma_y$ = 250 MPa, will carry a torque of 18 kNm. According to the von Mises yield criterion, what should the radius be to avoid yielding?

**A.** 38 mm
**B.** 40 mm
**C.** 43 mm
**D.** 45 mm
**E.** 49 mm

[2]

**SECTION B**
Answer ALL questions in this section

21. Fig. Q21 shows a semi-circular beam, with radius of curvature, $R$, that is built into the ground on an upper step. The other (free) end hangs over a lower step and is subjected to a load, $P$, at the tip.

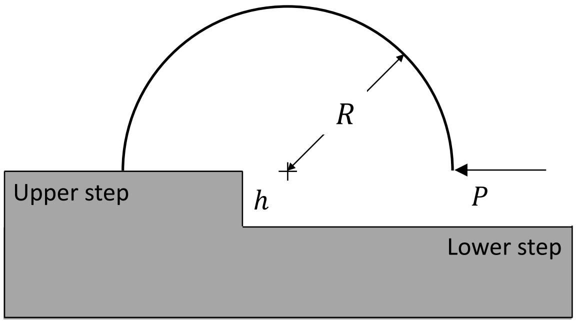


**Fig. Q21**

The beam is circular in cross section, with a diameter, $D$.

Using strain energy, determine an expression for the minimum step height, $h$, in terms of $P$, $R$, $E$ and $D$, if the free tip of the beam is allowed to only just make contact with the lower step.

The following trigonometric identity may be useful:

$$sin2\phi = 2sin\phi cos\phi$$

[20]

22. The cross-section of a straight I-section beam is shown in Fig. Q22. The beam will be considered unsafe if there is any plasticity in the web region.

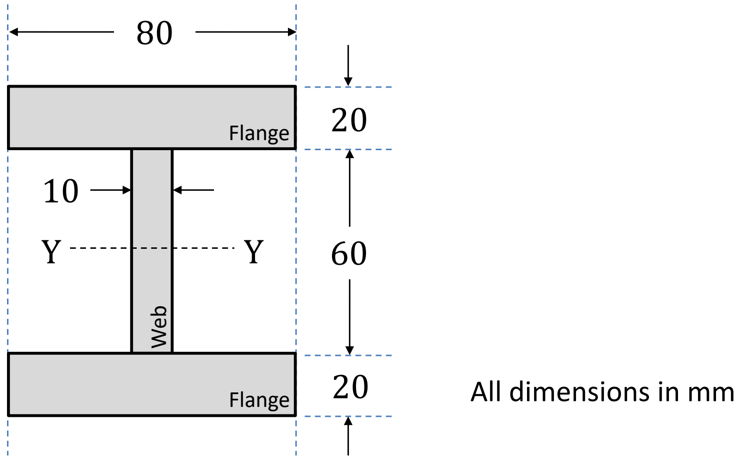


**Fig. Q22**

(a) After application of a pure bending moment, $M$, of $26.46\ \mathrm{kNm}$ about the Y-Y axis, the beam is deemed unsafe for the reason stated above. Show by calculation, that this is true, and determine the distance, $a$, from the Y-Y axis, that plasticity occurs. [12]

(b) Sketch the residual stress state in the beam when the bending moment is removed. [8]

The material can be assumed to be elastic-perfectly-plastic with a yield stress, $\sigma_y = 195\ \mathrm{MPa}$.

23. The section shown in Fig. Q23 carries a vertical shear force, $S = 32\ \text{kN}$ down the vertical centreline.

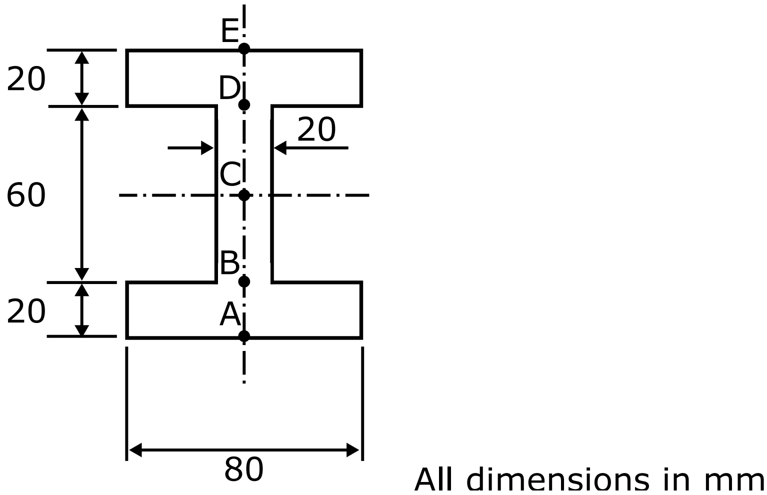


**Fig. Q23**

(a) Determine the vertical shear stress at points A, B, C, D and E. [16]

(b) Using the values calculated, sketch the shear stress distribution down the vertical centreline of the section. [4]

**END**